

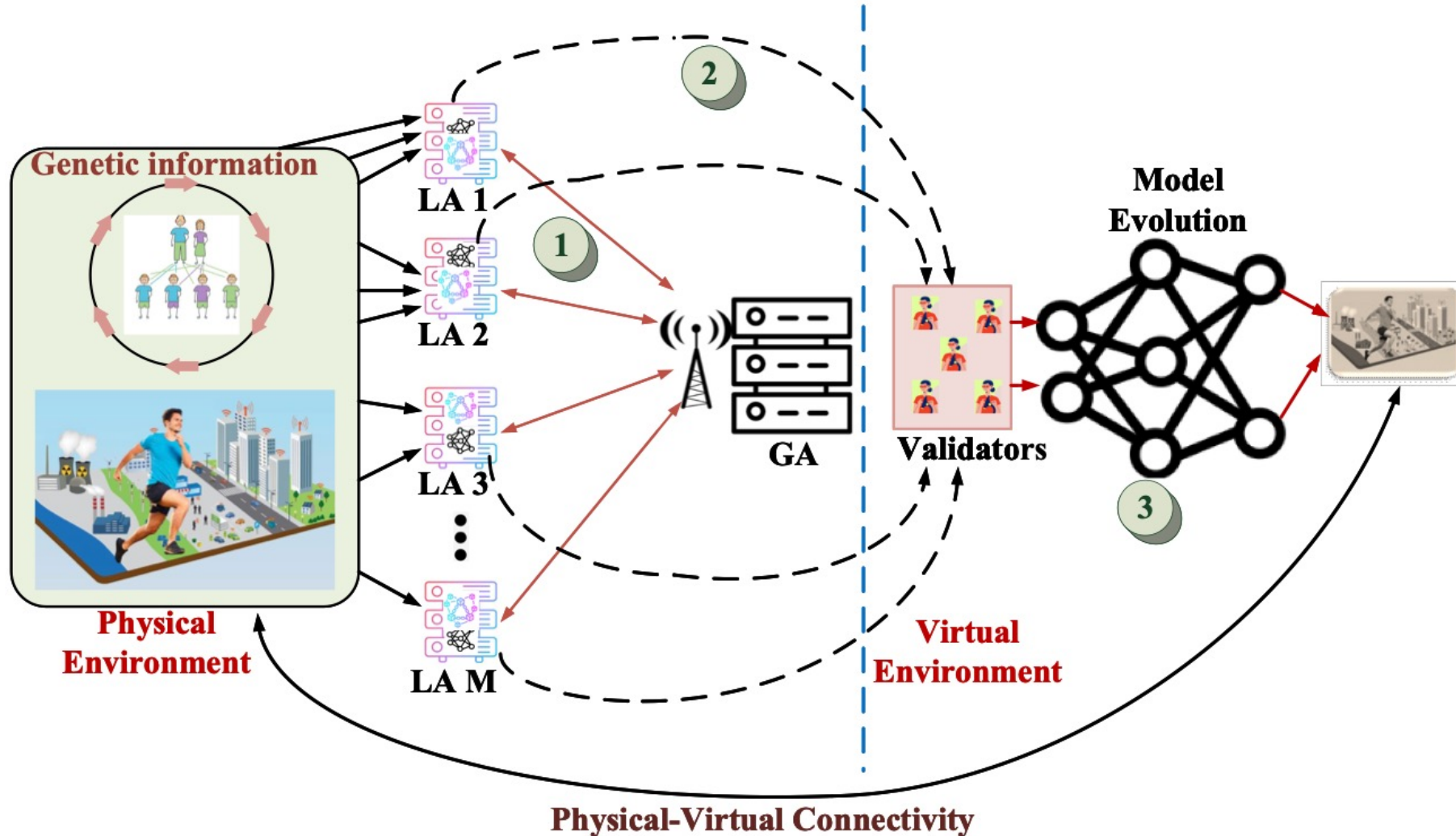
Differentially Private Federated Multi-Task Learning Framework for Enhancing Human-to-Virtual Connectivity in Human Digital Twin

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<https://ieeexplore.ieee.org/document/10234396>



Differentially Private Federated Multi-Task Learning Framework



Federated Multi-Task Learning

Feature Classifier

Domain classifier:

$$M_{MD}(i, GA) = \left\| \frac{1}{n_i} \sum_{k=1}^{n_i} \Phi(x_i^k) - \frac{1}{n_{GA}} \sum_{l=1}^{n_{GA}} \Phi(x_{M+1}^l) \right\|_{\mathcal{H}}^2$$

Logistic regression-based domain classifier:

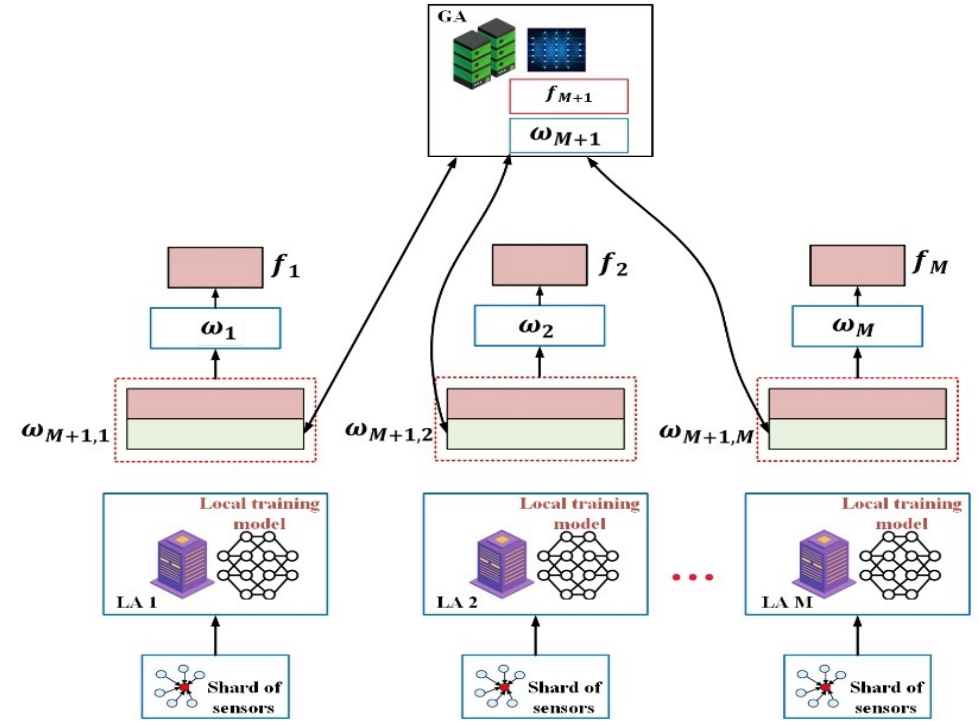
$$\min_{\Phi(i, GA)} \left\| \text{diag}(Q_i) \left(x_i^k - x_{M+1}^l \Phi(i, GA) \right) \right\|_F^2 + \lambda \sum_{l=1}^{n_{GA}} \left\| \Phi_l(i, GA) \right\|_F^2$$

$$\min_{\Phi(GA, i)} \left\| x_i^k - x_{M+1}^l \Phi(GA, i) \right\|_F^2 + \lambda \sum_{k=1}^{n_i} \left\| \Phi_k(GA, i) \right\|_F^2$$

FL Learning

Local training: $f_i(\omega_i, \omega_{M+1}) = \frac{1}{D_i} \sum_{j=1}^{D_i} \ell(x_{i,j}, y_{i,j}, \omega_i, \omega_{M+1})$

Global training: $f(\omega_{M+1}) = \sum_{i=1}^M \Omega_i f_i(\omega_{M+1,i}, \omega_{M+1})$



$(\epsilon, \delta) - DP$

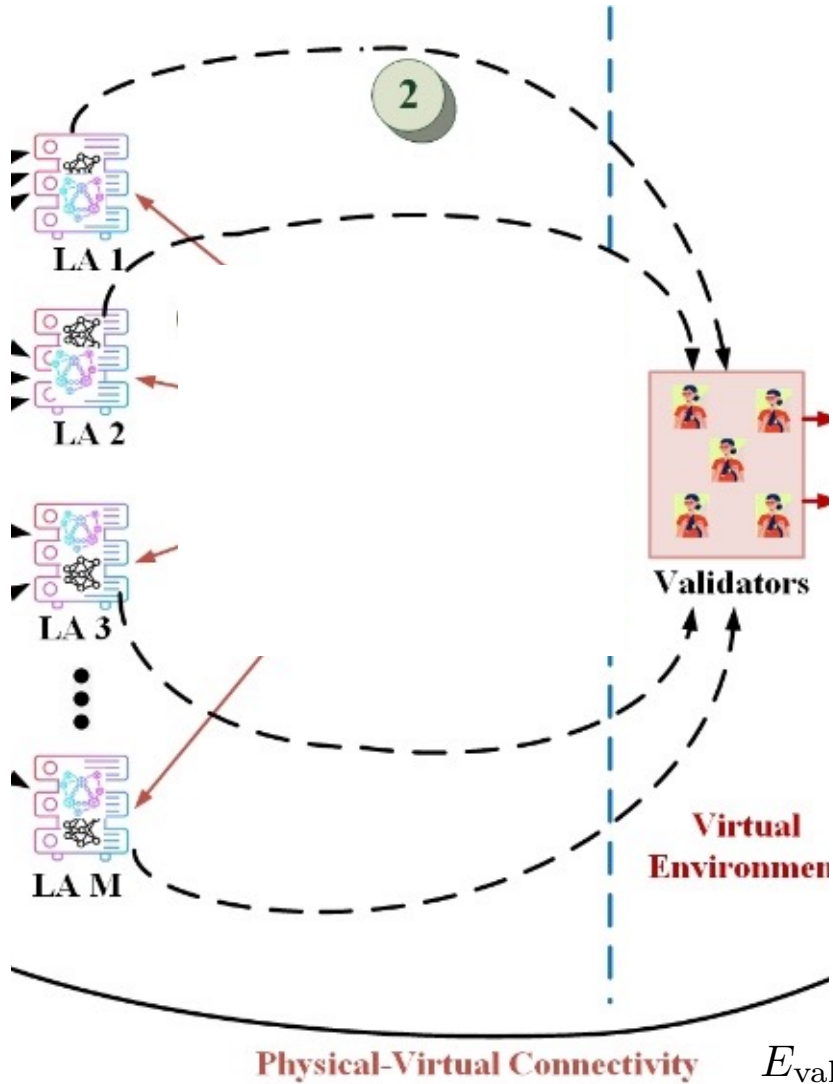
Differential Privacy (DP)

$$n_{M+1,i} \sim \mathcal{N}(0, \sigma^2 G_S^2)$$

Economical loss:

$$\psi = \frac{1}{\epsilon_{\max}} \sum_{i=1}^M \sum_{t=0}^{N_R-1} \epsilon_i v_i |\omega_{M+1,i}(t)|$$

Blockchain Based Validation Model



Proof of model quality (PoMQ)

$$R_{m_j}(i) = \begin{cases} 1 & \text{if } C_{\text{tot}}^{\text{off}}(i) \geq \theta_{\text{off}}, C_{\text{tot}}^{\text{cmp}}(i) \geq \theta_{\text{cmp}}, \text{ and } C_{\text{tot}}^{\text{pvy}}(i) \geq \theta_{\text{pvy}} \\ 0 & \text{otherwise,} \end{cases}$$

Validation time cost

$$T_{\text{val}}(i) = \underbrace{\frac{|\omega_{f,i}|}{r_i}}_{\text{Transmission time}} + \max_{m_j \in [V]} \left\{ \underbrace{\frac{c_v |R_{m_j}(i)|}{c_{m_i}}}_{\text{Computation time}} + \underbrace{\frac{|R_{m_j}(i)|}{r_v}}_{\text{Decision exchange time}} \right\}$$

Transmission time Computation time Decision exchange time

Validation energy cost

$$E_{\text{val}} = \frac{N}{P_i h_{i,\text{val}}} \left[\exp\left(\frac{r_i}{B_0} - 1\right) \right] + \sum_{m_j=1}^V \left\{ \frac{N}{P_j h_{j,k}} \left[\exp\left(\frac{r_v}{B_0} - 1\right) \right] + \kappa_v c_0 |R_{m_j}(i)| c_{m_i}^2 \right\}$$

Synchronization Accuracy

Synchronization Accuracy = Synchronization gap + FML loss

Synchronization gap = the time since the last status update

Proposition: If C_{time} is i.i.d. exponential in steady-state, the density of S_{gap} at any time t can be obtained as

$$\mathcal{D}_{S_{gap}}^{lcfs}(t) = \frac{o_i[(\rho + 2)(\rho - 1)]t - \rho^2 + \rho + 3}{\rho^3 - 1} \exp(-\rho_i t) + \frac{o_i(\rho + 1)t + \rho(\rho + 3) + 3}{\rho(\rho + 1) + 1} \exp(-\rho_i[\rho + 1]t) - \frac{\rho}{\rho - 1} \exp(-o_i t)$$

Synchronization gap

$$S_{gap}^{lcfs} = \frac{o_i^4(2o_i + 7\rho_i) + o_i^2\rho_i^2(8o_i + 7\rho_i) + \rho_i^4(4o_i + \rho_i)}{o_i\rho_i(o_i + \rho_i)^2(o_i^2 + \rho_i o_i + \rho_i^2)}$$

	Waiting on the queue	In service
Empty		
a arrives	→	a
b arrives	b	a
c arrives	c	a
a departs	→	c

Non-preemptive single-server last-com-first-serve (LCFS) queue with a buffer of size 2 and queue displacement policy

Connectivity Cost

Privacy cost

Long-term average connectivity cost:

$$C_{\text{conn}} = \lim_{\mathcal{U} \rightarrow \infty} \frac{1}{\mathcal{U}} \sum_{u=1}^{\mathcal{U}} \left(\frac{1}{c_{\text{max}}} \sum_{i=1}^M \sum_{t=0}^{N_R-1} \epsilon_i v_i |\omega_{M+1,i}^u(t)| + C_{\text{time}}^u + C_{\text{ene}}^u \right)$$

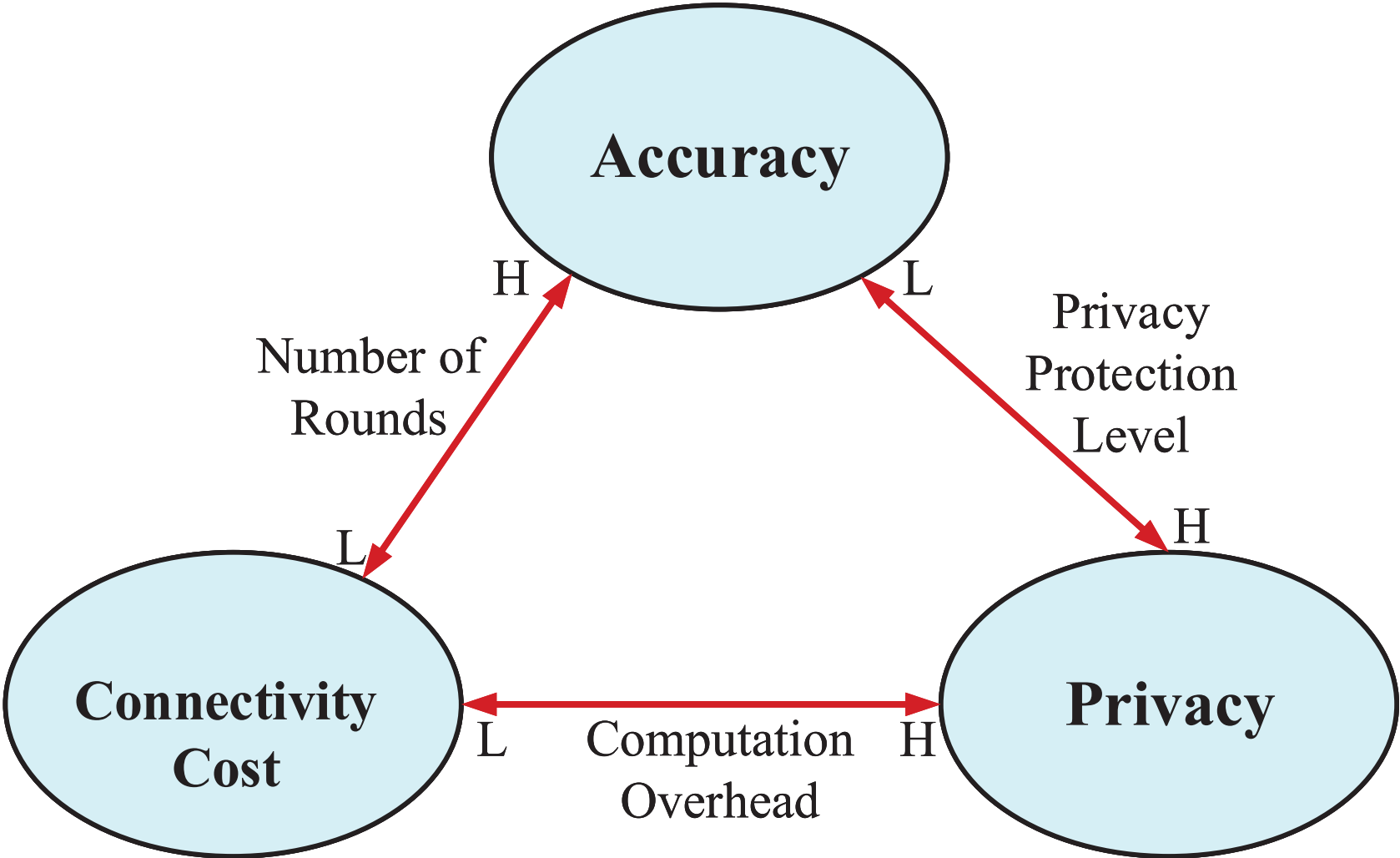
Overall time cost:

$$C_{\text{time}} = \sum_{t=0}^{N_R-1} \left\{ \left(\frac{c_{\text{agg}} \sum_{i=1}^M |\omega_{M+1,i}(t)|}{c_{GA}} \right) + \max_{i \in [M]} \left(\frac{|\omega_{M+1,i}|}{r_i} + \frac{c_r D_i(t)}{c_i} \right) \right\} \\ + \left(\frac{|\omega_{f,i}|}{r_i} + \max_{m_j \in [V]} \left\{ \frac{c_v |R_{m_j}(i)|}{c_{m_i}} + \frac{|R_{m_j}(i)|}{r_v} \right\} \right)$$

Overall energy cost:

$$C_{\text{ene}} = \sum_{t=0}^{N_R-1} \left\{ \kappa_{GA} c_0 \left(\sum_{i=1}^M |\omega_{M+1,i}(t)| c_{GA}^2 \right) + \sum_{i=1}^M \left(\kappa_i c_0 D_i(t) c_i^2 + t_i(t) \frac{N}{h_{i,GA}(t) P_i(t)} \left[\exp \left(\frac{r_i(t)}{B_0} - 1 \right) \right] \right) \right\} \\ + \frac{N}{P_i h_{i,\text{val}}} \left[\exp \left(\frac{r_i}{B_0} - 1 \right) \right] + \sum_{m_j=1}^V \left\{ \frac{N}{P_j h_{j,k}} \left[\exp \left(\frac{r_v}{B_0} - 1 \right) \right] + \kappa_v c_0 |R_{m_j}(i)| c_{m_i}^2 \right\}$$

Tradeoffs



Optimization Formulation

$$\min_{o_i, V, M, \epsilon, N_R} [\Theta_1 (C_{\text{time}} + C_{\text{ene}} + f_i(\omega_i, \omega_{M+1})) - (1 - \Theta_1) \Theta_2 \psi_{\text{cost}}],$$

s.t. $o_i \in [0, 1],$

$$\sum_{i \in [M]} i \leq M,$$
$$\sum_j^V m_j \leq V,$$
$$\epsilon_{\min} \leq \epsilon \leq \epsilon_{\max},$$
$$c_i^{\min} \leq c_i \leq c_i^{\max}, \forall i \in [M],$$
$$c_{m_j}^{\min} \leq c_{m_j} \leq c_{m_j}^{\max}, \forall m_j \in [V]$$

Overall time cost
Overall energy cost
Training accuracy
Privacy cost

o_i : status updating scheduling rate
 V : validators
 M : participating LAs
 ϵ : privacy budget
 N_R : FL rounds

DRL Solution

MDP Formulation

$$(\mathcal{S}^{(t)}, \mathcal{A}^{(t)}, \mathcal{R}^{(t)})$$

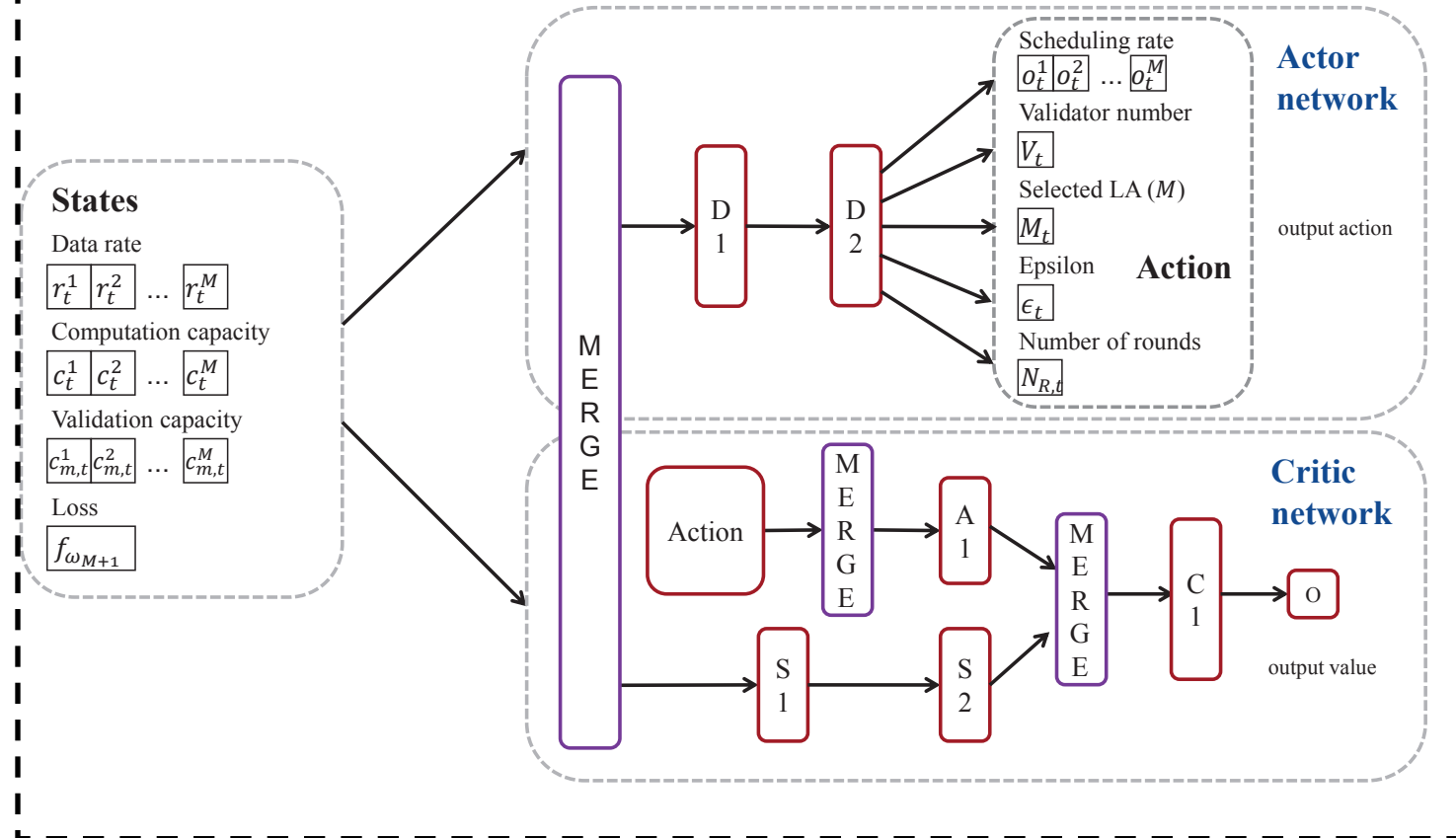
$$\mathcal{S}^{(t)} = \{r(t), c(t), c_m(t), f(\omega_{M+1}(t))\}$$

$$\mathcal{A}^{(t)} = \{o(t), V(t), M(t), \epsilon(t), N_R(t)\}$$

$$\mathcal{R}^{(t)} = -O(t)$$

$$\min \mathbb{E} \left[\sum_{t=0}^{N_R-1} \gamma \mathcal{R}(\mathcal{S}^{(t)}, \mathcal{A}^{(t)}) \right]$$

DDPG Solution



Simulation Results

FeDAvg: conventional FL

DPFedAvg: FeDAvg+DP

VDPFML: DPFML+PoS

